Delta Hedging

[Not shown in sample video]

…The process by which a market maker lessens the risk of holding inventory is called hedging, and delta hedging is a special version of that…

[Not shown in sample video]

…The clever way that a market maker can balance the two risks – delivery risk and position risk – is called delta hedging, which is a probabilistic hedging scheme.

Let's say the delta of the 60-strike call is 0.25 at initiation. This means there is a 25% chance of having to deliver 100 shares, or, from a statistical standpoint, 100% chance of having to deliver 25 shares. The market maker then buys 25 shares to probabilistically hedge delivery risk.

It is for this reason that delta is also known as the "hedge ratio" of an option position.

If the share price then increases to $60, the probability that the market maker will have to deliver 100 shares goes up to 50% because we know the delta is 0.50.

At this point, the market maker would buy an additional 25 shares for a total position of 50 at an average price of $55 / share.

If the share price goes up again after that, at expiration, the market maker would only have to buy 50 more shares to deliver the full 100 shares.

The average price the market maker pays to deliver the inventory is $62.50 and the price received is the $60 strike price and the $1.50 of premium. You can see that the loss on this hedging strategy compared to the simpleminded buy-inventory-at-the-end strategy is lower by 90%.

While it's not perfect in this case, you can see that the market maker has balanced the risk of not delivering the shares with the risk of being stuck with an inventory in the underlying.

There are a few things to point out about delta hedging... *[Not shown in sample video]*

Vocabulary

Delta Hedging: The process of creating a portfolio of related financial securities, in which the portfolio value remains unchanged when small changes occur in the value of the underlying security.

Gamma Risk: The risk that the move in price of an underlying security will be so large that it is impossible to adequately maintain one's delta hedge.

Hedge Ratio: This is another name for delta. For the mathematically inclined, the hedge ratio is given by the term $N(d_1)$ in the Black-Scholes equation.

Delta Hedging

Call option (C) and put option (P) prices are calculated using the following formulas:

$$C = S_0\, e^{-qt} * N(d_1) - X\, e^{-rt} * N(d_2)$$

$$P = X\, e^{-rt} * N(-d_2) - S_0\, e^{-qt} * N(-d_1)$$

... where $N(x)$ is the standard normal cumulative distribution function.

The formulas for $d1$ and $d2$ are:

$$d_1 = \frac{ln(\frac{S_0}{X}) + t\,(r - q + \frac{\sigma^2}{2})}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Where t = time to expiration, r = risk-free rate, q = dividend yield, σ = stock price variance

